coincide with DC. Draw AI parallel and AK perpendicular to DC and let $EFGH$ be the rectangle.

Then $\frac{1}{2}(AI+DC)\times AK=EF\times FG+FG\times GC+\frac{1}{2}(AI+EF)(AK-KR')$. But $AK=11.2$ inches, $AI=7.8$ inches. $\therefore$ $588=28\,EF+33FG$.

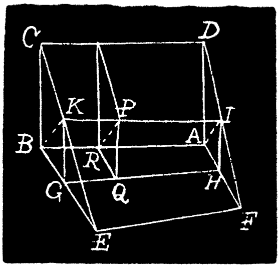
$\therefore$ for maximum $28\,EF=33\,FG$. $\therefore$ $EF=10.5$ inches, FG 8.91 inches.

26. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

$ABCD$ represents a triangle, and $ABEF$ a trapezoid which is perpendicular to the rectangle, both figures having the side AB common to each other. and ADF and BCE forming two right triangles perpendicular to the rectangle $ABCD$. To determine the conoidal surface $CDFE$ so as to satisfy the condition that any plane laid through AB will interest it in a straight line. Also find volume of the surface thus formed.

Solution by the PROPOSER.

Let $BC=AD=h$, $AB=a$, $BE=b$, $AF=c$. Let P represent a point in the surface, and put $AR=x$, $RQ=y$, $PQ=z$



The triangles BGK, PQR, and AHI are similar, and we may now put $AH=ny$, $IH=nz$, $BG=my$, $KG=mz$; but $h:mz=b-my$; $h:nz=c:c-nz$, whence $m=\dfrac{bh}{hy+bz}$, $n=\dfrac{ch}{hy+cz}$.

In the trapezoid $AHGB$, we now have $AB=a$, $AH=\dfrac{chy}{hy+cz}$, $BG=\dfrac{bhy}{hy+bz}$, $AR=x$, $RQ=y$. $\therefore$ $(AH+y)x+(BG+y)(a-x)=(AH+BG)a$.

Substituting, clearing of fractions, and arranging, we find for the equation of the surface

$$abcz^2+a(b+c)hyz-(b-c)h^2xy+ah^2y^2-abchz-ach^2y=0.$$

Let us now denote $\angle CBK=\angle DAI$ by θ, and angles BCK and ADI represent by C and D. For the volume we have $\frac{1}{6}ah^2\displaystyle\int_0^{\frac{\pi}{2}}\left[\frac{\sin^2 C}{\sin^2(C+\theta)}\right.$

$$\left.+\frac{\sin^2 D}{\sin^2(D+\theta)}+\frac{\sin C\sin D}{\sin(C+\theta)\sin(D+\theta)}\right]d\theta=\tfrac{1}{6}ah^2\left[\tan C+\tan D\right.$$

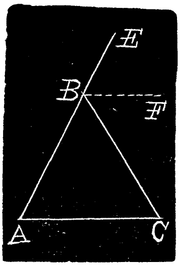
$$\left.+\frac{\tan C\tan D}{\tan C-\tan D}\log\frac{\tan C}{\tan D}\right]\text{ but }\tan C=\frac{b}{h},\ \tan D=\frac{c}{h};$$

$$\therefore\text{ volume}=\tfrac{1}{6}ah\left[b+c+\frac{bc}{b-c}\log\frac{b}{c}\right].$$

27. Proposed by ADOLPH BAILOFF, Durand Wisconsin.

A line BF, that bisects an angle exterior to the vertical angle of an isosceles triangle is parallel to the base AC.

Solution by Mrs. MARY E. HOGSETT, Danville, Kentucky: P. S. BERG. Apple Creek, Ohio, Professors JOHN FAUGHT, Bloomington, Indiana; and M. A. GRUBER, War Department, Washington, D. C.



$\angle EBC = \angle A + \angle C = 2\angle C$.

But $\angle EBC = 2\angle FBC$, since BF is the bisector of $\angle EBC$. $\therefore 2\angle FBC = 2\angle C$, or $\angle FBC = \angle C$.

Hence, BF is parallel to AC, because if two straight lnesi are cut by a third straight making the alternate interior angles equal, the two lines are parallel.

Solutions were also received from *J. K. ELLWOOD*, *H. G. WHITAKER*, and *G. B. M. JERR*.

NOTE—No solution has yet been received to problem 20.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

20. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

$$\int_0^{\frac{1}{2}\pi} \sqrt{[(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)]}\,d\phi = \text{what?}$$

Solution by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

$$\sqrt{(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)} = \sqrt{1-e^2(\cos^2\phi+\sin^2\phi)+e^4\sin^2\phi\cos^2\phi}$$

$$=\sqrt{1-e^2+\frac{e^4}{4}\sin^2 2\phi} = \sqrt{1-e^2+\frac{e^2}{4}-\frac{e^4}{4}\cos^2 2\phi}$$

$$=\tfrac{1}{2}\sqrt{(2-e^2)^2-e^4\cos^2 2\phi} = \tfrac{1}{2}\sqrt{(2-e^2)^2-e^4\sin^2\left(\frac{\pi}{2}-2\phi\right)}$$

$$=\tfrac{1}{2}(2-e^2)^2\sqrt{1-\frac{e^4}{(2-e^2)^2}\sin^2\left(\frac{\pi}{2}-2\phi\right)}.$$

$$\therefore \int_0^{\frac{\pi}{2}} d\phi\sqrt{(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)}$$

$$=-\tfrac{1}{4}(2-e^2)\int_{\frac{1}{2}\pi}^{-\frac{1}{2}\pi} d\left(\frac{\pi}{2}-2\phi\right)\sqrt{1-\frac{e^4}{(2-e^2)^2}\sin^2\left(\frac{\pi}{2}-2\phi\right)}$$

$$=\tfrac{1}{4}(2-e^2)\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi} d\lambda\sqrt{1-\frac{e^4}{(2-e^2)^2}\sin^2\lambda}$$